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Ultimate strain analysis of rock and soil foundation of underground tunnel

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Abstract. In order to improve effectiveness of ultimate strain analysis of geotechnical materials for foundation of underground tunnel, an ultimate strain analysis method of geotechnical materials for foundation of underground tunnel based on discretization finite element method of phase change and displacement field was proposed. Firstly, strain analysis model of geotechnical materials for foundation of underground tunnel under fully saturated state has porous matrix and crack characteristics; secondly, a discretization finite element method was proposed. It realized simulated calculation process design of phase change by using dynamic solving of phase field and displacement field; finally, effectiveness of proposed algorithm was verified by empirical analysis.

Key words. Phase change, Displacement field, Finite element, Geotechnical materials, Ultimate strain.

1. Introduction

Mechanical coupling problem of geotechnical materials for foundation of underground tunnel is the problem [1, 2] encountered frequently in earth science field. As injecting or removing change of pore pressure formed by geology of geotechnical materials, geotechnical deformation can be caused. For example, how to avoid sedimentation caused by excessive underground water pumping is very important in use and management of underground water. At the same time, similar problems also exist in the collection process of petroleum and natural gas. For example, coupling flow mechanical model can be used to determine safe injection pressure to avoid damaging integrity of geotechnical material cover in the process of CO2 sequestration. Especially geomechanical effect in formation of fissured geotechnical materials is more important. Key point of the work is to study numerical solution method of fully fluid-solid coupled mechanical model of large fracturing formation. Interest

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point studied here is to predicted and pre-existing formation behavior [3] of crack and fault.

Two main aspects studied by such problems are flow of geotechnical materials and geotechnical mechanics. As geometric correlation is complex, solution adopted in the Thesis is simulation based on discretization finite element of phase change and displacement field. There have been many mature technologies for flow simulation of discrete crack of porous matrix. For example, classical finite element technology can be used to implement discrete simulation [4] for multiple phase flow of cracks of geotechnical materials in the frame of finite element. There are mixed finite element method [5], discontinuous finite element method [6] and so on afterwards. Mixed pressure equation and finite volume transmission equation of finite volume method can be adopted for research [7] in finite volume frame.

Finite element and boundary element method are frequently adopted for analysis in mechanical analysis of geotechnical materials. Finite volume method and flow mechanical coupling modeling method also have mature application. Different from seepage flow model, cracks have heterogeneity in the aspect of permeability, so mechanical behavior of fissured rock mass is more complex. Solid contact deformation problem has been widely studied. Different approximate technologies can be used, such as finite element, extended finite element and so on. Realizing stable discretization scheme of non-linear contact problems is relatively difficult [9]. Stability of classic methods such as Lagrange multiplier method and regularization penalty method is poor. Stabilization equation of contact mechanics problem is obtained by using parameter stabilization in Literature [10]. A Nitsche handling method is proposed in Literature [11]. The method can be regarded as uniform penalty function method. When model parameter selection is proper, corresponding discrete equation set usually has better adjustability.

Flow and mechanical coupling behaviors in geotechnical materials are usually non-linear and hydraulic conductivity and frictional resistance are available to simulate crack sliding. Crack model of discrete flow-solid coupling mechanics has already been proposed in Literatures at present. Different from method in the Thesis, it takes no account of contact problem of crack. For example, contact problem of fracture surface is not taken into account in Literature [12] and fracture characteristics are taken into account in Literature [13]. One grid is used to represent formation of unstructured grid of discrete crack here in the work. Different strategies can be used to solve coupling flow and mechanical problem. Complete implicit equation is used to solve non-linear equation, boundary conditions and related constitutive relation of coupling system in the work. Crack and fault are expressed in large scope or effective characteristics of fissured rock mss are calculated in small scope.

2. Mathematical mode

Mathematical mode proposed here describes the behavior that porous geotechnical materials under fully saturated state contain a set of fractures. In order to reach the purpose of modeling, two areas are divided: porous matrix and crack, and a method that can compress single-phase geotechnical material is proposed.

Crack is defined as two planes contacting mutually. Mechanical model of cracks of geotechnical materials must be described through superficial stress of crack and displacement field change. Unstressed contact [14] between surfaces of two geotechnical materials is described in Fig. 1.

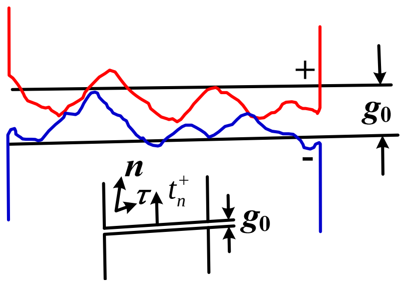

Fig. 1. Non stress contact cracks in shale

g_0 is characteristic distance width of crack in Fig. 1. When $g_0 = 0$, it is corresponding to "ideal" contact status. Relative position of the surface can be defined by gap function $g = (g_N, g_T)$. Normal component g_N is defined as relative displacement related to reference distance g_0. Tangential component represents length of displacement vector in the fracture surface of geotechnical materials. Normal gap g_N has three different configurations: (1) $g_N = 0$ represents unstressed free contact; (2) $g_N > 0$ represents penetration phenomenon caused by deformation of contact points; (3) $g_N < 0$ represents no physical contact.

Traction t in the surface can be defined as mapping of Terzaghi effective stress tensor σ' on normal component n. Local coordinate system is used to obtain:

$$t = -\sigma' n = t_N n + t_T \tau \,. \tag{1}$$

Traction $(+, -)$ can be defined for two surfaces of track of geotechnical materials respectively; the following equation can be obtained:

$$\begin{cases} t_N^+ = -t_N^- = t_N \,, \\ t_T^+ = -t_T^- = t_T \,. \end{cases} \tag{2}$$

If two surfaces lose physical contact, crack surface of geotechnical materials is separated by pressure of geotechnical materials. $n \cdot \sigma \cdot n = -p$ at this time. Therefore, $t_N = 0$. Dynamic behavior of crack can be described by traction determined by gap function:

$$\begin{cases} t_N = \mathcal{N}(g_N) \\ t_T = \mathcal{F}(t_N), g_T \neq 0 \\ t_T < \mathcal{F}(t_N), g_T = 0 \end{cases} \tag{3}$$

Where, $g_T \neq 0$ represents sliding state and $g_T = 0$ represents sticky state. For negative normal gap, crack surface loses physical contact. Fracture flow capacity can be evaluated based on the following models:

$$C_f = -\frac{g_N^3}{12} + C_{f,0} \,. \tag{4}$$

3. Finite element calculation

3.1. Discretization finite element

Variation of total energy functional of sum of three energies of volume chemical energy, interface energy and elastic strain energy is solved; the following equation can be obtained:

$$\frac{\delta G_{tot}}{\delta \eta_p} = \frac{\partial[\varphi_{chem}(\eta_p, T) + \varphi_e(\varepsilon_{ij}, \eta_p, T)]}{\partial \eta_p} - \frac{d}{dx_i}\left(\frac{\partial \varphi_{int}(\eta_{p,i})}{\partial \eta_{p,i}}\right) \quad p = 1, 2, 3 ... n \,. \tag{5}$$

Put equation (3-1)into Ginzburg-Landau equation of equation(2-1), the following equation can be obtained:

$$\frac{\partial \eta_p}{\partial t} - L\beta \nabla \cdot \nabla \eta_p = -L\frac{\partial(\varphi_{stn} + \varphi_{chem})}{\partial \eta_p} + \xi_p \quad p = 1, 2, 3 ... n \,. \tag{6}$$

Equation (6) is similar to unstable control equation of temperature field. Continuous C0 is only needed to be guaranteed and regular finite element interpolation is only needed to be adopted in finite element discretization. Variable order parameter η in phase field shall be discrete with finite element and node variable shall be adopted to represent order parameter η. Equation (6) shall be discrete, and the following equation can be obtained:

$$[C_\eta]\frac{\partial\{\eta_k\}}{\partial t} + [K_\eta]\{\eta_k\} = \{Q_k(\eta_1, \eta_2, ... \eta_n, \{\sigma\})\} \quad k = 1, 2, 3 ... n \,. \tag{7}$$

3.2. Dynamic solving of phase field

The relation between order parameter at the time of ti+1 and at the time of ti in phase field is expressed by the following equation:

$$\eta_{i+1} = \eta_i + (1-\beta)\Delta t \dot{\eta}_i + \beta \Delta t \dot{\eta}_{i+1} \,. \tag{8}$$

Put the above recursion equation into phase field equation after discretization; the following equation can be obtained after arrangement

$$[\hat{K}]\{\eta_p\}_{i+1} = \{\hat{Q}_p\} \,. \tag{9}$$

Where equivalent stiffness matrix and equivalent right-hand item are:

$$[\hat{K}] = [C] + \Delta t \beta [K_c] \,. \tag{10}$$

$$\{\hat{Q}_p\} = (1-\beta)\Delta t\{Q_p\}_i + \beta\Delta t\{Q_p\}_{i+1} + ([C] - (1-\beta)\Delta t[K_c])\{\eta_p\}_i \,. \tag{11}$$

The maximum time step of the solving method with numerical stability is:

$$\Delta t = \frac{2}{(1-2\beta)\lambda_{\max}} \,. \tag{12}$$

λ is the maximum eigenvalue of equation$([K_c] - \lambda[C])\{\bar{T}\} = \{0\}$.

When β=0, the method is explicit integration method of time domain. Equation (11) and equation (12) can be transformed into

$$[\hat{K}] = [C] \,. \tag{13}$$

$$\{\hat{Q}_k\} = \Delta t\{Q_k\}_i + ([C] - \Delta t[K_c])\{\eta_k\}_i \,. \tag{14}$$

3.3. Dynamic solving of displacement field

(1) Newmark-β method:

Relation between displacement, speed and accelerated speed at the time of ti+1 and displacement, speed and accelerated speed at the time of ti in Newmark-β method can be expressed by the following equation:

$$\dot{u}_{i+1} = \dot{u}_i + (1-\gamma)\Delta t\ddot{u}_i + \gamma\Delta t\ddot{u}_{i+1} \,. \tag{15}$$

$$u_{i+1} = u_i + \Delta t\dot{u}_i + (\frac{1}{2} - \beta)\Delta t^2\ddot{u}_i + \beta\Delta t^2\ddot{u}_{i+1} \,. \tag{16}$$

Arrange dynamic equation of displacement field of recursion equation of equation (15) and equation (16), and then step-by-step integration equation can be obtained:

$$\begin{cases} [\hat{K}]\{u\}_{i+1} = \{\hat{P}\}_{i+1} \,, \\ \{\dot{u}\}_{i+1} = \dfrac{\gamma}{\beta\Delta t}(\{u\}_{i+1} - \{u\}_i) + (1 - \dfrac{\gamma}{\beta})\{\dot{u}\}_i + \Delta t(1 - \dfrac{\gamma}{2\beta})\{\ddot{u}\}_i \\ \{\ddot{u}\}_{i+1} = \dfrac{1}{\beta\Delta t^2}(\{u\}_{i+1} - \{u\}_i) - \dfrac{1}{\beta\Delta t}\{\dot{u}\}_i - (\dfrac{1}{2\beta} - 1)\{\ddot{u}\}_i \,. \end{cases} \tag{17}$$

Where equivalent stiffness matrix and equivalent load vector are respectively:

$$[\hat{K}] = [K] + \frac{1}{\beta\Delta t^2}[M] + \frac{\gamma}{\beta\Delta t}[C] \,. \tag{18}$$

$$\begin{aligned} \{\hat{P}\}_{i+1} =& \{R\}_{i+1} + [M][\frac{1}{\beta\Delta t^2}\{u\}_i + \frac{1}{\beta\Delta t}\{\dot{u}\}_i + (\frac{1}{2\beta} - 1)\{\ddot{u}\}_i] \\ &+ [C][\frac{\gamma}{\beta\Delta t}\{u\}_i + (\frac{\gamma}{\beta} - 1)\{\dot{u}\}_i + \frac{\Delta t}{2}(\frac{\gamma}{\beta} - 2)\{\ddot{u}\}_i] \,. \end{aligned} \tag{19}$$

Stability condition of Newmark-β method is:

$$\Delta t \leq \frac{1}{\pi\sqrt{2}} \frac{1}{\sqrt{\gamma - 2\beta}} T_n \,. \tag{20}$$

When $\gamma = 1/2$ and $\beta = 1/4$, algorithm is unconditionally stable and has second order accuracy at the same time. Relation among speed and accelerated speed at the time of *ti,* and displacement at the time of ti+1 and ti-1 in central difference method can be expressed by the following equation:

$$\dot{u}_i = \frac{u_{i+1} - u_{i-1}}{2\Delta t} \,. \tag{21}$$

$$\ddot{u}_i = \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta t^2} \,. \tag{22}$$

Put recursion equation into force balance equation and the following equation can be obtained:

$$[\hat{K}]\{u\}_{i+1} = \{\hat{P}\}_{i+1} \,. \tag{23}$$

Where:

$$[\hat{K}] = \frac{1}{\Delta t^2}[M] + \frac{1}{2\Delta t}[C] \,. \tag{24}$$

$$\{\hat{P}\}_{i+1} = \{R\}_i - ([K] - \frac{2}{\Delta t^2}[M])\{u\}_i - (\frac{1}{\Delta t^2}[M] - \frac{1}{2\Delta t}[C])\{u\}_{i-1} \,. \tag{25}$$

Stability condition of central difference method is:

$$\Delta t \leq \frac{T_n}{\pi} \,. \tag{26}$$

3.4. Simulated calculation flow of phase change

As for coupling problem of displacement field and phase field, adopted solution strategy is to solve displacement field at the time of t firstly according to distribution value of order parameter at the time of$t$,and then solve phase field at the time of$t+\Delta t$ according to obtained stress distribution value at the time of t to obtain distribution value of order parameter at the time of $t + \Delta t$. Specific flow is shown in Fig. 2.

4. Experimental analysis

Numerical experiment is used for verification in our analytical solution in the section. We take Mandel problem without crack into account under the first condition. Purely broken mechanical problem is used for verification of crack model under the second condition.

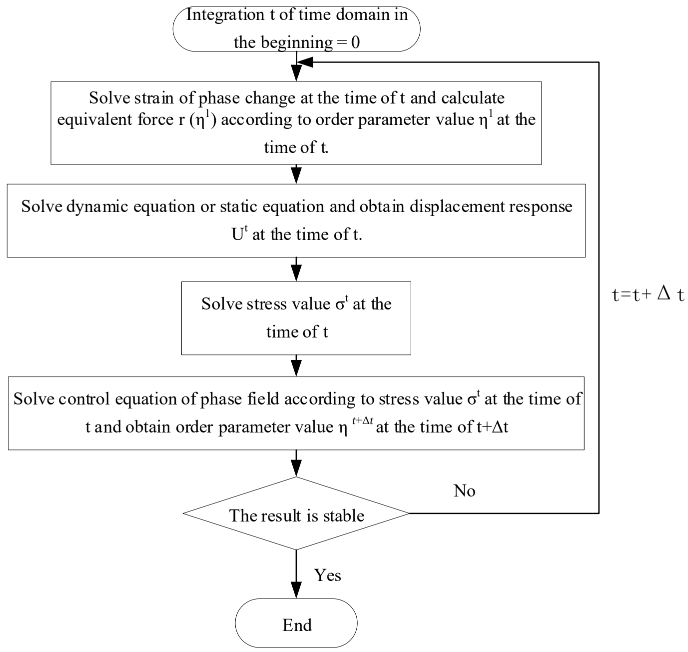

Fig. 2. Simulated calculation flow diagram of phase change

4.1. Mandel problem

Considering two-dimensional Mandel problem, Mandel problem is usually used to evaluate solving accuracy of flow and geomechanical coupling. Problem setting is described in Fig. 3a. Domain is defined as:

$$\Omega = [0, a] \times [0, b] = [0, 20m] \times [0, 10m] . \tag{27}$$

Mechanical boundary condition is left boundary ($u_x|_{x=0} = 0$) of zero horizontal displacement. Zero vertical displacement of lower boundary is ($u_z|_{z=0} = 0$) and uniform load of upper boundary is $F|_{z=b} = 500 \times l0^5 Pa$. In addition, displacement constraint condition of upper boundary is $u_z\, |\forall x$ = constant. The domain is completely saturated single-phase geotechnical material, compressibility is $c = 2 \times 10^{-10} Pa^{-1}$, viscosity is $\mu_f = 2 \times 10^{-3} Pa$, reference density is $P_f = 1000\ kg/m^3$, and initial pressure of geotechnical material is $p\,\big|\Omega = 80.35 \times 10^5 Pa$. Assuming that Young modulus of porous geotechnical material is $E = 2 \times 10^{10} Pa$, undrained or drained Poisson ratio is 0.393 and 0.2 respectively and Biot modulus is 1. Reference porosity is 0.2 and matrix permeability is 2 millidarcies.

Undrained Poisson ratio is adopted for initialization and drained Poisson ratio is adopted to simulate transient behavior of system in the system. Comparison with transient pressure of analytical solution value calculation is shown in Fig. 3b, and pressure in the position of ($x = 0, z = 0$) is selected for comparison. Proposed numerical value model can catch transient behavior of pressure solution accurately.

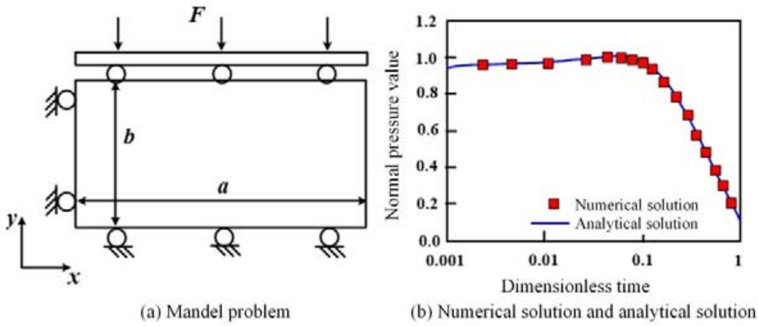

Fig. 3. Numerical and analytical solutions of Mandel problem

4.2. Single crack problem

Friction sliding of single crack is considered in two-dimensional setting. Analytical solution and numerical solution of the mechanical problem can be obtained. Single crack will receive constant stress in infinite medium, $\sigma_\infty = -350 \times 10^5 Pa$. Young modulus and Poisson ratio of medium is $E = 2 \times 10^{10} Pa$ and $v = 0.25$ respectively. Frictional angel can be defined as $\beta = 30^0$. Crack length and angle of inclination is respectively: $2b = 10m$ and $\alpha = 20^0$. Penalty value is $\varepsilon_N = 1 \times 10^{11}$. First-order and second-order tetrahedral element can be used for three-dimensional discretization structure of the problem. Analytical expression t_N of traction, stress σ_T and tangential clearance can be respectively calculated into:

$$\begin{cases} T_N = -\sigma_\infty \sin^2(\alpha) \ , \\ \sigma_T = \sigma_\infty \sin(\alpha)\cos(\alpha) - \sigma_\infty \sin^2(\alpha)\tan(\alpha) \ , \\ E g_T = 4\sigma_T \left(1 - v^2\right)\sqrt{b^2 - (x-b)^2} \ . \end{cases} \tag{28}$$

Comparison of analytical solution and numerical solution of several grid resolutions is shown in Fig. 4. As for displacement, the scheme is pretty good. As for t_N, analytical solution is constant value. Quality of numerical solution deteriorates in the surrounding of crack. It is a well-known vertex singularity problem of numerical solution of stress. Fig. 4b shows that increased resolution ratio and approximation order can realize improvement of numerical solution. Special shape function in crack top can be used for improvement.

5. Conclusions

An ultimate strain analysis method of geotechnical materials for foundation of underground tunnel based on discretization finite element method of phase change and displacement field was proposed in the Thesis to realize stress behavior analysis of porous geotechnical materials under completely saturated condition. Effectiveness of proposed method was verified by production scene experiment of fully coupled

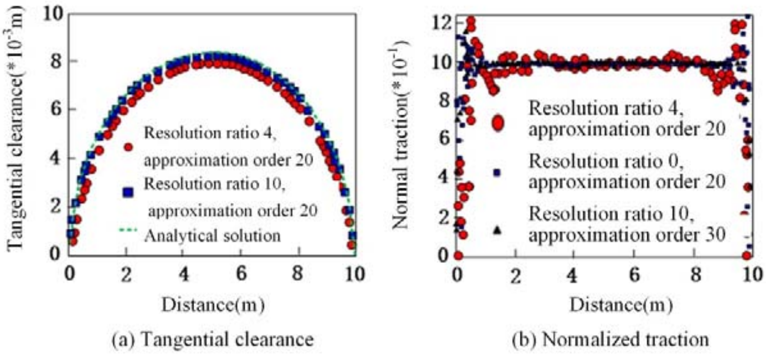

Fig. 4. Analytical solution and numerical solution

and nonlinear equation set and geotechnical materials. Research emphasis of the next step will be focused on: (1) solver performance analysis of non-linear negative influence related to cracks; (2) non-linear solver design of large-scale crack model; (3) influence of model initialization on solving process and so on.

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References

[1] X. H. Li, H. T. Wang, J. Q. Jia, et al.: *Ultimate displacement discrimination of stability and reliability analysis of surrounding rocks of tunnel and underground engineering*[J]. Rock & Soil Mechanics, *26* (2005), No. 6, 850–854.

[2] J. A. Wang, H. D. Park: *Comprehensive prediction of rockburst based on analysis of strain energy in rocks*[J]. Tunnelling & Underground Space Technology, *16* (2001), No. 1, 49–57.

[3] Y. K. Lee, S. Pietruszczak: *A new numerical procedure for elasto-plastic analysis of a circular opening excavated in a strain-softening rock mass*[J]. Tunnelling & Underground Space Technology, *23* (2008), No. 5, 588–599.

[4] H. B. Fan, Z. H. Fan, Z. L. Feng: *Lose Stability Analysis of Underground Tunnel under Strong Shock Rock Plastic Strain*[J]. Chinese Quarterly of Mechanics, *23* (2002), No. 1, 105–108.

[5] L. R. Alejano, A. Rodríguez-Dono, M. Veiga: *Plastic radii and longitudinal deformation profiles of tunnels excavated in strain-softening rock masses*[J]. Tunnelling and Underground Space Technology, *30* (2012), No. 3, 169–182.

[6] L. Song, H. Z. Li, C. L. Chan, et al.: *Reliability analysis of underground excavation in elastic-strain-softening rock mass*[J]. Tunnelling & Underground Space Technology, *60* (2016), 66–79.

[7] J. A. Wang, H. D. Park: *Comprehensive prediction of rockburst based on analysis of strain energy in rocks*[J]. Tunnelling & Underground Space Technology, *16* (2001), No. 1, 49–57.

[8] V. N. Zhukov: *On Variations of the Stress-Strain State in the Rock Mass During Construction of Underground Chambers for Machine Rooms of Hydroelectric and Pumped-*

Storage Power Plants[J]. Soil Mechanics & Foundation Engineering, *38* (2001), No. 4, 120–124.

[9] Y. Fan, W. B. Lu, P. Yan, et al.: *Mechanism of strain energy adjustment of surrounding rock during excavation of underground caverns*[J]. Yantu Lixue/rock & Soil Mechanics, *34* (2013), No. 4, 3580–3586.

[10] L. R. Alejano, A. Rodríguez-Dono, M. Veiga: *Plastic radii and longitudinal deformation profiles of tunnels excavated in strain-softening rock masses*[J]. Tunnelling and Underground Space Technology, *30* (2012), No. 4, 169–182.

[11] L. Frgić, K. Tor, A. Jaguljnjak-Lazarević: *The inverse estimate for the excavation stability around a tunnel*[J]. Tunnelling and Underground Space Technology incorporating Trenchless Technology Research, *21* (2006), No. 3, 369–369.

[12] Y. Wang, X. H. Zeng, J. M. Yin, et al.: *Numerical simulation of zonal cracking of deep tunnel surrounding rock considering unloading effect*[J]. Yantu Lixue/rock & Soil Mechanics, *33* (2012), No. 4, 1233–1239.

[13] C. Paraskevopoulou, M. Diederichs: *A comparison of viscous models under constant strain and constant stress: Implications for tunnel analysis*[C]// Underground - the Way To the Future: Proceedings of the World Tunnel Congress, Wtc. (2013), 2013, 723–730.

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